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Radiative corrections in grand unified theories based on N=1 supergravity

I. Non-gauge theories

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ABSTRACT

Radiative corrections in grand unified theories based on N=1 supergravity are studied in the limit $M_{Planck} \to \infty$ with the gravitino mass m_g fixed, to all orders in perturbation theory. In this paper we study the effect of non-gauge interactions only. It is shown that the tree level mass hierarchy is not destroyed by the radiative corrections, provided there is no light field of mass $<< M_{GUT}$, which transforms as a singlet under the subgroup that is unbroken at a scale above m_g. It is also pointed out that in the minimal supersymmetric SU(5) model, the addition of arbitrary soft supersymmetry breaking terms in the lagrangian do not, in general, preserve the mass hierarchy, unless the coefficients of the soft breaking terms are fine tuned. The fine tuning is automatic if the supersymmetry breaking terms come from an underlying supergravity theory.

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I. Introduction

Supersymmetry provides a potential solution to the hierarchy problem[1] which led many authors to construct grand unified models based on supersymmetric theories[2]. However, since supersymmetry is not an observed symmetry of nature, it must be broken at a scale of order 100 GeV or more. On the other hand, if we w ant to use supersymmetry to explain the smallness of the $SU(2)^{weak}\times U(1)$ breaking scale compared to the Planck scale or the grand unification scale, the supersymmetry breaking scale should not be much higher than about a TeV, at least in the observable sector containing all the known fields of the low energy theory. An elegant way of breaking supersymmetry, which is consistent with most of the phenomenological constraints, is based on models with N=1 supergravity coupled to vector and chiral superfields[3]. In most of these models the superfields are divided into two classes, the hidden sector and the observable sector. The hidden sector contains only gauge singlet chiral superfields, whereas the observable sector contains all the gauge fields, the gauge non-singlet chiral superfields, and may also contain some gauge singlet chiral superfields. The total superpotential is taken to be the sum of two terms W_H and W_o where W_H is a function of the fields in the hidden sector only, and W_o is a function of the fields in the observable sector only. If ϕ_h and ϕ_i denote the chiral superfields in the hidden and the observable sectors respectively, and z_h and z_i denote their scalar components, the tree level potential involving the scalar fields is given by[4],

$$V = \exp\{8\pi G(\sum_h |z_h|^2 + \sum_i |z_i|^2)\}$$

$$\Big[\sum_h \Big| \frac{\partial W_H}{\partial z_h} + 8\pi G z_h^*(W_0+W_H)\Big|^2 + \sum_i \Big|\frac{\partial W_0}{\partial z_i} + 8\pi G z_i^*(W_0+W_H)\Big|^2$$

$$- 24\pi G |W_0+W_H|^2\Big]$$

$$+ \frac{e^2}{2}\sum_\alpha \Big|\sum_{i,j} z_i^*(T_\alpha)_{ij} z_j\Big|^2 \qquad (1.1)$$

where G is the constant of gravitation ($1/M_p^2$) and the sum over a runs over all generators of the gauge group. The superpotential W_H of the hidden sector is assumed to have the form,

$$W_H(z_h) = \mu^3 f(z_h/M_p) \qquad (1.2)$$

where M_p is the Planck mass and μ is a mass parameter of order 10^{12}-10^{13}GeV. In the absence of the observable sector fields, the potential involving the hidden sector fields is assumed to have a minimum at $z_h = z_h^{(0)}$, such that $W_H(z_h^{(0)}) \sim \mu^3$. This breaks supersymmetry spontaneously, giving the gravitino a mass,

$$m_g = \exp(4\pi G \sum_h |z_h^{(0)}|^2)\; 8\pi G\, W_H(z_h^{(0)}) \qquad (1.3)$$

which is of order 100 GeV. We take m_g to be real for simplicity.

In analyzing the full theory we must minimize the full potential with respect to all the fields to determine the ground state. The radiative corrections in this theory will involve the fields in the hidden sector as well as the observable sector, and also the non-renormalizable interactions mediated by gravitons and gravitinos. However, the theory is greatly simplified if we study it in the limit $\mu\to\infty$, $M_p\to\infty$, with m_g, and the grand unification mass M (which sets the scale of the observable sector superpotential W_0) fixed. In this limit, the z_h fields are frozen at the minimum of the potential obtained from W_H, and the effective potential involving the fields in the observable sector is given by,

$$V = \sum_i \left|\frac{\partial W}{\partial z_i} + m_g z_i^*\right|^2 + \{m_g (A-3) W(z) + h.c.\} + D\text{-terms} \qquad (1.4)$$

where,

$$W(z) = \exp\{4\pi G \sum_h |z_h^{(0)}|^2\}\ W_0(z) \qquad (1.5)$$

$$A = \left[\sum_h z_h \frac{\partial W_H}{\partial z_h} + 8\pi G\, W_H \sum_h |z_h|^2\right]^*_{z_h = z_h^{(0)}} \Big/ W_H(z_h^{(0)}) \qquad (1.6)$$

In real life, however μ is smaller than M, and it is not a priori clear that the effective potential V, given in (1.4) has any connection with reality. But as we shall discuss now, the effective potential given in (1.4) is of interest due to several reasons.

i) Hall, Lykken and Weinberg[5] analyzed the problem in a somewhat different way. They assumed that in the limit of global supersymmetry, when the hidden and the observable sector fields are completely decoupled, the observable sector contains a set of heavy fields with masses of order $M(\sim 10^{16}\mathrm{GeV})$ and a set of massless fields z_α. They then eliminated the fields in the hidden sector, as well as the heavy fields in the observable sector by minimizing the full potential (1.1) with respect to these fields, and obtained an effective potential involving the light fields only. The effective potential was found to be independent of the heavy scales M and M_p, and depends only on the scale m_g. Hence the fields which were massless in the supersymmetric limit, acquire a mass of order $m_g \sim 100$ GeV, and the hierarchy of mass scales is not destroyed.

We may also start from the theory described by the potential (1.4) and eliminate the heavy fields by minimizing the potential with respect to these fields. As we shall show in Sec.II. the effective potential found this way is identical to the effective potential found in Ref.5, except for corrections of order M/M_p in the value of m_g . Hence, at least at the tree level, the effective potential (1.4) is as good as the full potential (1.1) in finding out the low energy predictions of the theory.[6]

ii) The full tree level potential given in (1.1) may be expressed as a sum of the potential given in (1.4) and some extra terms, which involve coupling between the hidden sector fields z_h and the observable sector fields z_i, and also terms which involve only the observable sector fields z_i or only the hidden sector fields z_h. If we take m_g, M and M_p to be the three independent mass scales of the theory, then all these extra interaction terms will have explicit powers of M_p in the denominator. Hence it is unlikely that any radiative correction involving these vertices will cancel the radiative corrections involving only the vertices given in (1.4). Hence a necessary, but certainly not sufficient, condition for the stability of mass hierarchy under radiative corrections in the full theory is that the radiative corrections in the truncated theory described by the superpotential (1.4) will not affect the mass hierarchy present at the tree level of the theory.

Hence in this paper we shall analyze the effect of radiative corrections in the theory described by the potential (1.4). The theory may be described as a globally supersymmetric theory described by the superpotential $W(\phi_i)$ with explicit soft supersymmetry breaking terms in the action of the form,

$$-\left\{ m_g \sum_i z_i \frac{\partial W(z)}{\partial z_i} + m_g (A-3) W(z) + h.c. \right\} - m_g^2 \sum_i |z_i|^2 \qquad (1.7)$$

In order to illustrate the subtleties involved in the analysis of the problem, let us consider minimal supersymmetric SU(5) model, given by the superpotential,

$$W(\Phi, H, \tilde{H}) = \lambda_1\{Tr(\Phi^3) + M_1\, Tr(\Phi^2)\} + M_2\, \tilde{H}_i\, H_i + \lambda_2\, \tilde{H}_i\, \Phi_{ij}\, H_j \tag{1.8}$$

where Φ, H and $\tilde{H}$ belong to the 24, 5 and $\bar{5}$ representations of SU(5) respectively. In the limit of unbroken supersymmetry, the potential is given by,

$$V = \sum_{i,j} |\, \lambda_1\{3(\Phi^2 - \tfrac{1}{5} Tr\, \Phi^2)_{ij} + 2M_1\, \Phi_{ij}\} + \lambda_2(H_i \tilde{H}_j - \tfrac{1}{5} H_k \tilde{H}_k\, \delta_{ij})|^2$$

$$+ \sum_i |\sum_j (\lambda_2\, \Phi_{ij} + M_2\, \delta_{ij})\, H_j|^2 + \sum_j |\sum_i \tilde{H}_i\, (\lambda_2\, \Phi_{ij} + M_2\, \delta_{ij})|^2$$

$$+ D \text{ terms} \tag{1.9}$$

The potential has a minimum at,

$$\langle\Phi\rangle = \frac{4}{3} M_1 \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & -3/2 & \\ & & & & -3/2 \end{pmatrix}$$

$$\langle H_i \rangle = \langle \tilde{H}_i \rangle = 0 \tag{1.10}$$

The masses of the weak doublet and the color triplet higgses are respectively,

$$m_2 = -2M_1\lambda_2 + M_2 \qquad\qquad m_3 = \frac{4}{3} M_1 \lambda_2 + M_2 \tag{1.11}$$

and by fine tuning the parameters so that $M_2=2M_1\lambda_2+O(m_g)$, the higgs doublet mass may be kept much smaller than its color triplet partner.

Let us now consider the effect of adding an arbitrary soft supersymmetry breaking term in the Lagrangian, e.g.

$$-m_g \sum_{i,j} [A\lambda_2 \Phi_{ij} \tilde{H}_i H_j + (A-1) M_2 \tilde{H}_i \delta_{ij} H_j] \qquad (1.12)$$

which are two of the terms in the soft breaking terms given in (1.7). If we now minimize the full potential keeping $M_2=2M_1\lambda_2$, the soft breaking term (1.12) will produce a large mass term of the form $-m_gM_2H\tilde{H}$ for the higgs doublet. By retuning the parameters M_1 and M_2, we may keep the mass of one particular linear combination of H and $\tilde{H}^\dagger$ to be of order m_g, but the orthogonal linear combination will have a mass of order $\sqrt{m_gM}$. When we include the effect of radiative corrections, the mass terms proportional to $|H|^2$ and $|\tilde{H}|^2$ get renormalized, and hence both the higgs fields acquire mass of order $\sqrt{m_gM}$, unless we readjust the parameters of the theory. Hence the individual soft breaking terms given in (1.7) may destroy the hierarchy of mass scales which is present, in the supersymmetric limit. However, as we shall show in Sec.II, due to a delicate cancellation between different terms in (1.7), any mass hierarchy present in the supersymmetric limit is unperturbed by the soft supersymmetry breaking terms given in (1.7).

In Sec.III we shall study the effect of radiative corrections in the theory. In this paper, we only analyze theories without gauge interactions. In the limit of unbroken supersymmetry, no-renormalization theorems guarantee that the radiative corrections do not destroy the tree level mass hierarchy. Various authors[7,8] have studied the effect of supersymmetry breaking on mass hierarchy in globally and locally supersymmetric models. But there is no general proof of the stability of the mass hierarchy against radiative corrections to all orders in perturbation theory. In this paper we shall analyze the problem by writing down the most general effective potential in the superfield formalism. This is done by using a powerful theorem due to Grisaru, Rocek and Siegel[9,10]. We show that to all orders in perturbation theory the radiative corrections generated due to the soft supersymmetry breaking terms do not destroy the hierarchy of mass scales, provided there is no light field of mass of order m_g in the theory which transforms as a singlet under the unbroken symmetry group of the theory. The instability of mass hierarchy in the presence of light singlet fields has been discussed previously by several authors[7]. We summarize our result in Sec.IV.

II. LOW ENERGY EFFECTIVE POTENTIAL AT THE TREE LEVEL

We shall consider a supersymmetric model with superpotential $W(\phi)$, which is assumed to be invariant under some symmetry group G. We shall denote by z_i the scalar components of the superfields ϕ_i. If supersymmetry is unbroken, the potential involving the scalar fields is given by,

$$V_0(z_i) = \sum_i \left| \frac{\partial W}{\partial z_i} \right|^2 \tag{2.1}$$

In this paper we shall ignore the effect of all gauge interactions, and take G to be a global symmetry group. The potential (2.1) is assumed to have a supersymmetric minimum at $z_i = z_i^{(0)}$, where,

$$\frac{\partial W(z)}{\partial z_i} = 0 \qquad \forall\, i \tag{2.2}$$

Some of the scalar fields are assumed to have a non-zero vacuum expectation value (vev) of order M at this minimum, which breaks the group G to one of its subgroups H. We shall divide the scalar fields into the following two classes:

z_A: These are superheavy complex scalars which acquire masses of order M at the minimum of the potential.

z_α: These are light complex scalars which are massless in the supersymmetric limit. In a realistic model, z_α's include all the known fields of mass$\lesssim$1TeV. Also, since in the present discussion G is a global symmetry group, the Goldstone bosons corresponding to the symmetry breaking, and the scalar fields belonging to the same supermultiplet as these Goldstone bosons, are masless and included in the set z_α. In the presence of gauge interactions the Goldstone bosons are absorbed by the gauge bosons through the higgs mechanism, whereas their partners, belonging to the same supermultiplet, acquire large mass through the D term of the potential and become degenerate with the gauge bosons. These were considered as a separate class of fields in the analysis of Ref.5.

As can be seen from Eq.(2.1), the mass^2 matrix at the minimum of the potential is given by the square of the matrix $(\partial^2 W/\partial z_i \partial z_j)|_{z=z^{(0)}}$. Since z_A and z_α denote the heavy and the light fields respectively, we have at $z=z^{(0)}$,

$$\frac{\partial^2 W}{\partial z_\alpha \partial z_\beta} = \frac{\partial^2 W}{\partial z_\alpha \partial z_A} = 0 \qquad (2.3)$$

$$\frac{\partial^2 W}{\partial z_A \partial z_B} = M_{AB} \sim M \qquad (2.4)$$

For reasons that will become clear later, we shall assume that none of the light fields z_α is a singlet under the unbroken subgroup H. Hence,

$$z_\alpha^{(0)} = 0 \tag{2.5}$$

Let us now introduce explicit soft supersymmetry breaking terms in the potential of the form,

$$\Delta V = \left\{ m_g (A-3)\, W + m_g \sum_i z_i \frac{\partial W}{\partial z_i} + h.c. \right\} + m_g^2 \sum_i |z_i|^2 \tag{2.6}$$

where m_g is the gravitino mass, and A is a constant of order unity. The origin of these terms has been discussed in the introduction. The total potential is then given by,

$$V = V_0 + \Delta V \tag{2.7}$$

We shall eliminate the heavy fields z_A from the potential by minimizing the potential with respect to these fields, and construct an effective potential involving the light fields z_α. Following Ref.5 we calculate the effective potential to order m_g^4 for arbitrary values of the fields z_α of order m_g. Let us express the value of z_A at the minimum of V, for arbitrary values of the fields $z_\alpha \sim m_g$, as,

$$z_A = z_A^{(0)} + z_A^{(1)} + z_A^{(2)} + \cdots \tag{2.8}$$

where $z_A^{(n)}$ is of order $m_g^{\,n}/M^{n-1}$. In writing down (2.8), we have implicitly assumed that $z_A - z_A^{(0)}$ is at most of order m_g. This assumption may be verified at the end of the calculation. Using Eqs. (2.1), (2.6) and (2.7), we get,

$$\frac{\partial V}{\partial z_B} = \sum_{i=\alpha, A} \frac{\partial^2 W}{\partial z_B \partial z_i} \left(\frac{\partial W}{\partial z_i} + m_g z_i^* \right)^* + m_g \left(\frac{\partial W}{\partial z_B} + m_g z_B^* \right)$$
$$+ m_g (A-3) \frac{\partial W}{\partial z_B} = 0 \qquad (2.9)$$

which may be written as,

$$\sum_A \frac{\partial^2 W}{\partial z_B \partial z_A} \left(\frac{\partial W}{\partial z_A} + m_g z_A^* \right)^*$$
$$= - \sum_\alpha \frac{\partial^2 W}{\partial z_B \partial z_\alpha} \left(\frac{\partial W}{\partial z_\alpha} + m_g z_\alpha^* \right)^* - m_g \left(\frac{\partial W}{\partial z_B} + m_g z_B^* \right)$$
$$- m_g (A-3) \frac{\partial W}{\partial z_B} \qquad (2.10)$$

Assuming that $z_A - z_A^{(0)} \sim m_g$, $z_\alpha \sim m_g$, and using Eqs.(2.3), (2.4), we may easily show that the first term on the right hand side of (2.10) is of order $m_g^{\,3}$, whereas the second and the third terms are at most of order $m_g^{\,2}M$. On the other hand, $(\partial^2 W/\partial z_B \partial z_A)$ is a non-singular matrix with eigenvalues of order M. Hence from Eq.(2.10) we get,

$$\frac{\partial W}{\partial z_A} + m_g z_A^* \sim m_g^{\,2} \qquad (2.11)$$

Substituting this on the right hand side of Eq.(2.10) we get,

$$\frac{\partial W}{\partial z_A} + m_g z_A^* = m_g^2 (A-3) \quad (M^{-1})_{AB} z_B^{(0)*} + O(m_g^3/M) \tag{2.12}$$

which gives,

$$\left(\frac{\partial^2 W}{\partial z_A \partial z_B}\right)_0 (z_B^{(1)} + z_B^{(2)}) + \frac{1}{2}\left(\frac{\partial^3 W}{\partial z_A \partial z_B \partial z_C}\right) z_B^{(1)} z_C^{(1)}$$

$$+ \frac{1}{2}\left(\frac{\partial^3 W}{\partial z_A \partial z_\alpha \partial z_\beta}\right) z_\alpha z_\beta + \left(\frac{\partial^3 W}{\partial z_A \partial z_B \partial z_\alpha}\right) z_B^{(1)} z_\alpha + m_g (z_A^{(0)*} + z_A^{(1)*})$$

$$= m_g^2 (A-3) (M^{-1})_{AB} z_B^{(0)*} + O(m_g^3/M) \tag{2.13}$$

where repeated indices are summed over. Equating terms of order m_g M on both sides we get,

$$z_B^{(1)} = - m_g (M^{-1})_{BA} z_A^{(0)*} \tag{2.14}$$

Equating terms of order m_g^2 on both sides of (2.13), we get,

$$z_B^{(2)} = (M^{-1})_{BA} \Big\{ m_g^2 (A-3)(M^{-1})_{AC} z_C^{(0)*} - m_g z_A^{(1)*}$$

$$- \frac{1}{2}\left(\frac{\partial^3 W}{\partial z_A \partial z_C \partial z_D}\right) z_C^{(1)} z_D^{(1)} - \left(\frac{\partial^3 W}{\partial z_A \partial z_C \partial z_\alpha}\right) z_C^{(1)} z_\alpha$$

$$- \frac{1}{2}\left(\frac{\partial^3 W}{\partial z_A \partial z_\alpha \partial z_\beta}\right) z_\alpha z_\beta \Big\} \tag{2.15}$$

We may now proceed to evaluate the full effective potential. Eqs.(2.1), (2.6) and (2.7) give,

$$V = \sum_A \left| \frac{\partial W}{\partial z_A} + m_g z_A^* \right|^2 + \sum_\alpha \left| \frac{\partial W}{\partial z_\alpha} + m_g z_\alpha^* \right|^2$$
$$+ \{ m_g (A-3)\, W(z_A, z_\alpha) + h.c. \} \tag{2.16}$$

From (2.12) we see that $\partial W/\partial z_A + m_g z_A^*$ is of order m_g^2, and independent of the light fields z_α. Hence the first term of (2.16) contributes just a constant term to $V_{eff}(z_\alpha)$ to order m_g^4. The second term in (2.16) may be written as,

$$\sum_\alpha \left| \frac{\partial W_{eff}}{\partial z_\alpha} + m_g z_\alpha^* \right|^2 \tag{2.17}$$

where,

$$W_{eff}(z_\alpha) = W(z_A = z_A^{(0)} + z_A^{(1)}, z_\alpha) - W(z_A = z_A^{(0)} + z_A^{(1)}, z_\alpha = 0) \tag{2.18}$$

Using Eq.(2.3) we may show that W_{eff} is of order m_g^3, and hence (2.17) is of order m_g^4. Also since $z_A^{(1)}$ is independent of z_α, $\partial W_{eff}/\partial z_\alpha$ is equal to $\partial W/\partial z_\alpha$ to order m_g^2. This shows the equality of (2.17) and the second term of (2.16).

Finally the third term on the right hand side of (2.16) is given to order m_g^4 by,

$$m_g (A-3) \{ (W(z) - W_{eff}(z_\alpha)) + W_{eff}(z_\alpha) \} + h.c.$$

$$= [\, m_g (A-3) \{ W(z_A, z_\alpha) - W(z_A = z_A^{(0)} + z_A^{(1)}, z_\alpha) \} + m_g (A-3) W_{eff}(z_\alpha)]$$

$$+ h.c. + \text{terms independent of } z_\alpha \qquad (2.19)$$

By expanding both $W(z_A, z_\alpha)$ and $W(z_A=z_A^{(0)}+z_A^{(1)}, z_\alpha)$ about the point $z_i=z_i^{(0)}$, we may reduce (2.19) to,

$$m_g (A-3) W_{eff}(z_\alpha) + m_g (A-3) \left(\frac{\partial^2 W}{\partial z_A \partial z_B} \right)_0 z_A^{(1)} z_B^{(2)} + h.c.$$

$$+ \text{terms independent of } z_\alpha + O(m_g^5/M) \qquad (2.20)$$

Using Eqs.(2.15) and dropping the constant terms (remember that $z_A^{(1)}$ is independent of z_α) we express (2.20) as,

$$m_g (A-3) W_{eff}(z_\alpha) - m_g (A-3) \left\{ \frac{1}{2} \left(\frac{\partial^3 W}{\partial z_A \partial z_\alpha \partial z_\beta} \right)_0 z_A^{(1)} z_\alpha z_\beta \right.$$

$$\left. + \left(\frac{\partial^3 W}{\partial z_A \partial z_B \partial z_\alpha} \right)_0 z_A^{(1)} z_B^{(1)} z_\alpha \right\} + h.c.$$

$$\equiv m_g (A-3) \{ W_{eff}(z_\alpha) - W_{eff}^{(2)}(z_\alpha) - 2 W_{eff}^{(1)}(z_\alpha) \} + h.c. \qquad (2.21)$$

where $W_{eff}^{(2)}$ and $W_{eff}^{(1)}$ are respectively the terms in W_{eff} quadratic and linear in z_α's. (2.21), together with (2.17) gives the full effective potential of the system,

$$V_{eff} = \sum_{\alpha} \left| \frac{\partial W_{eff}}{\partial z_{\alpha}} + m_g z_{\alpha}^{*} \right|^2 + [\, m_g (A-3) \{ W_{eff} (z_{\alpha})$$
$$- W_{eff}^{(2)} (z_{\alpha}) - 2 W_{eff}^{(1)} (z_{\alpha}) \} + h.c.] \qquad (2.22)$$

This is identical to the expression for the effective potential derived in Ref.5.

We may illustrate the significance of the above results by considering the SU(5) model introduced in Sec.I. According to the results derived in this section, the weak doublet parts of the H, $\tilde{H}$ fields acquire a mass at most of order m_g after the introduction of the soft supersymmetry breaking terms. What really happens is that after we introduce the supersymmetry breaking terms, the quantity $\lambda_1 [3(\phi^2 - \mathrm{Tr}\,\phi^2/5) + 2M_1\phi]$ no longer vanishes at the minimum of the potential, instead it is of order $m_g M$. The first term on the right hand side of Eq.(1.9) then gives rise to a mass term of the H, $\tilde{H}$ fields of the form $m_g M H\tilde{H}$. This term exactly cancels the term given in (1.12). This cancellation requires that the various supersymmetry breaking terms have their coefficients as given in Eq.(1.7), and a small deviation from this form will produce a large mass of the weak doublet higgs.

III. RADIATIVE CORRECTIONS

In this section we shall discuss the effects of radiative corrections in the class of models discussed in Sec.II using the superfield formulation. Let ϕ_i denote the chiral superfields of the theory. Let us also introduce a spurion superfield η, defined as,

$$\eta = m_g \theta^2 \tag{3.1}$$

where θ , $\bar{\theta}$ are the fermionic coordinates of the superspace. The Lagrangian density of the theory introduced in Sec.II may then be written as,

$$\begin{aligned}
&\left\{ \int d^2\theta \, W(\phi) + h.c. \right\} + \int d^2\theta \, d^2\bar{\theta} \; \bar{\Phi}_i \; \phi_i \\
&- \left\{ \int d^2\theta \left(\eta \, \phi_i \frac{\partial W}{\partial \phi_i} + \eta \, (A-3) \, W(\phi) \right) + h.c. \right\} \\
&- \int d^2\theta \; d^2\bar{\theta} \;\; \bar{\eta} \, \eta \;\; \bar{\Phi}_i \;\; \phi_i
\end{aligned} \tag{3.2}$$

where the terms involving η are the explicit soft supersymmetry breaking terms. The potential involving the scalar fields obtained from the Lagrangian density (3.2) may be written as,

$$\begin{aligned}
&- \sum_i F_i^{\dagger} F_i - \sum_i \left(F_i \frac{\partial W}{\partial z_i} + h.c. \right) \\
&+ \left[m_g \left\{ \sum_i z_i \frac{\partial W}{\partial z_i} + (A-3) \, W(z) \right\} + h.c. \right] + m_g^{\,2} \sum_i |z_i|^2
\end{aligned} \tag{3.3}$$

where F_i's are the auxiliary components of the superfields. Elimination of F_i's through their equations of motion gives the potential $V_0+\Delta V$ given in Eqs.(2.1) and (2.6).

Before proceeding further, we shall make some rearrangement of various terms. Let $z_i^{(c)}$ denote the point at which the potential (3.3) has its minimum with respect to all the fields z_A and z_α. We may then consider perturbation expansion around the point $z_i^{(c)}$. Let us define,

$$\hat{z}_i = z_i - z_i^{(c)} \tag{3.4}$$

By straightforward algebraic manipulation, the full potential (3.3) may be brought into the form,

$$\begin{aligned} &-\sum_i (F_i - m_g z_i^{(c)})^\dagger (F_i - m_g z_i^{(c)}) \\ &-\sum_i \{(F_i - m_g z_i^{(c)})(\frac{\partial W}{\partial z_i} + m_g z_i^{(c)*}) + h.c.\} \\ &+ m_g \{\sum_i \hat{z}_i \frac{\partial W}{\partial z_i} + (A-3) W(z) + h.c.\} \\ &+ m_g{}^2(\sum_i |z_i|^2 - \sum_i |z_i^{(c)}|^2) \end{aligned} \tag{3.5}$$

Let us define,

$$\tilde{F}_i = F_i - m_g z_i^{(c)} \tag{3.6}$$

$$\tilde{W}(\hat{z}) = W(z) + m_g z_i^{(c)*} \hat{z}_i \tag{3.7}$$

Then the potential (3.5) may be written as

$$- \sum_i \tilde{F}_i^{+} \cdot \tilde{F}_i - \sum_i \left(\tilde{F}_i \frac{\partial \tilde{W}}{\partial \hat{z}_i} + h.c. \right)$$

$$+ m_g \left\{ \sum_i \hat{z}_i \frac{\partial W}{\partial z_i} + (A-3)\, W(z) + h.c. \right\}$$

$$+ m_g^{\,2} \sum_i |z_i|^2 + \text{constant terms} \tag{3.8}$$

The first two terms in (3.8) may be interpreted as coming from a supersymmetric theory with superpotential $\tilde{W}$. The other terms are explicit soft breaking terms in the model. Since $\tilde{W}$ and W differ from each other by a linear term, they give rise to the same scalar-fermion Yukawa coupling. If ϕ_i denote the superfields of the original model, then we define,

$$\hat{\phi}_i = \phi_i - z_i^{(e)} \tag{3.9}$$

The action may then be written as,

$$\left\{ \int d^2\theta\, \tilde{W}(\hat{\phi}) + h.c. \right\} + \int d^4\theta\, \bar{\hat{\phi}}_i \hat{\phi}_i$$

$$- \left\{ \int d^2\theta\, \eta \left(\hat{\phi}_i \frac{\partial W}{\partial \phi_i} + (A-3)\, W(\phi) \right) + h.c. \right\}$$

$$- \int d^2\theta\, d^2\bar{\theta}\, \bar{\eta}\, \eta\, \bar{\phi}_i \phi_i \tag{3.10}$$

The structure of radiative corrections in this theory is constrained by a theorem due to Grisaru, Rocek and Siegel[9]. The theorem states that the effective action generated by radiative corrections in the theory must be of the form,

$$\int \left(\prod_j d^4 x_j\right) \int d^4\theta \quad f\left(\Phi_i(x_j, \theta), \eta(\theta)\right) \qquad (3.11)$$

where f is a polynomial in the superfields Φ_i, $\bar{\Phi}_i$, η, $\bar{\eta}$ and their covariant derivatives, but does not contain any explicit dependence on θ or $\bar{\theta}$. Note that the effective action, although a non-local function of the x_j's, is a local function of θ, since it involves product of fields at the same fermionic coordinates $\theta, \bar{\theta}$. This theorem, together with the rules of integration over the anti-commuting numbers c,

$$\int dc = 0 \qquad \int c\, dc = 1 \qquad (3.12)$$

restricts the possible radiative corrections in the theory. Since the θ and $\bar{\theta}$ integrals in (3.11) must be saturated according to the rule of integration given in (3.12), the possible radiatively induced terms must be of the form $\tilde{F}_i^\dagger \tilde{F}_j$, $\tilde{F}_i^\dagger \eta$, $\eta^\dagger \tilde{F}_i$, $\eta^\dagger \eta$, or terms involving higher powers of $\tilde{F}_i$ and/or η[11], multiplied by arbitrary functions of the scalar fields. Hence we may write the radiatively induced effective potential as,

$$-\tilde{F}_j^{\dagger}\tilde{F}_i\, h_{ij}(\hat{z},\hat{z}^{\dagger}) - m_g\{\tilde{F}_i\ g_i(\hat{z},\hat{z}^{\dagger}) + h.c.\}$$

$$+ m_g^2\ f(\hat{z},\hat{z}^{\dagger}) + O(\tilde{F}^3) \qquad (3.13)$$

where f, g and h are polynomials in the scalar components of the superfields. For the time being, we shall ignore the $O(\tilde{F}^3)$ term in the effective potential. It will be argued later that these terms do not destroy the mass hierarchy.

Taking the full potential as the sum of (3.8) and (3.13), and eliminating the $\tilde{F}_i$ fields through their equations of motion, we get,

$$V = \left\{ \frac{\partial\tilde{W}}{\partial\hat{z}_i} + m_g\, g_i(\hat{z},\hat{z}^{\dagger})\right\}^{*} (I+h)^{-1}_{ij} \left\{ \frac{\partial\tilde{W}}{\partial\hat{z}_j} + m_g\, g_j(\hat{z},\hat{z}^{\dagger})\right\}$$

$$+ m_g \left\{ \sum_i \hat{z}_i \frac{\partial W}{\partial\hat{z}_i} + (A-3)\, W(z) + h.c.\right\}$$

$$+ m_g^2 \left\{ \sum_i |z_i|^2 + f(\hat{z},\hat{z}^{\dagger})\right\} \qquad (3.14)$$

dropping the constant terms in the potential.

The functions f, g_i and h_{ij} have mass dimensions 2, 1 and 0 respectively. Hence, in general, the functions h, g, f, $\partial g_i/\partial z_j$ and $\partial f/\partial z_i$ will at most be of order unity, M, M^2, unity and M respectively, upto factors of logM and $\log M_p$.

This is true unless the graphs contributing to these functions have power law infrared divergence in the $m_g \to 0$ limit, in which case we may get extra powers of M/m_g. It can be shown that such infra-red divergences are absent in the class of theories being analyzed here[12].

Besides satisfying the bounds estimated from pure dimensional analysis, $\partial f/\partial z_\alpha$ and g_α satisfy the bounds,

$$|g_\alpha| \lesssim m_g \qquad\qquad \left|\frac{\partial f}{\partial z_\alpha}\right| \lesssim m_g \tag{3.15}$$

due to the following reason. Since the potential must be invariant under the unbroken subgroup H of G, and none of the fields $z_{\alpha'}$ transform as singlets under H (as we have assumed) both g_α and $\partial f/\partial z_\alpha$ must transform non-trivially under H. Hence both of them vanish at $z_{\alpha'}=0$, and must be of order $z_{\alpha'}\sim m_g$ for non-zero values of $z_{\alpha'}$.

We now minimize the potential with respect to the fields z_A (or $\hat{z}_A$). This gives,

$$\sum_{i,j} \Big[\Big(\frac{\partial^2 \tilde{W}}{\partial \hat{z}_j \, \partial \hat{z}_A} + m_g \frac{\partial g_j}{\partial \hat{z}_A} \Big) (I+h)^{-1}_{ij} \Big(\frac{\partial \tilde{W}}{\partial \hat{z}_i} + m_g g_i(\hat{z}, \hat{z}^\dagger) \Big)^*$$

$$+ m_g \frac{\partial g_i^*}{\partial \hat{z}_A} (I+h)^{-1}_{ij} \Big(\frac{\partial \tilde{W}}{\partial \hat{z}_j} + m_g g_j \Big)$$

$$+ \Big(\frac{\partial \tilde{W}}{\partial \hat{z}_i} + m_g g_i \Big)^* \frac{\partial}{\partial \hat{z}_A} \{ (I+h)^{-1}_{ij} \} \Big(\frac{\partial \tilde{W}}{\partial \hat{z}_j} + m_g g_j \Big) \Big]$$

$$+ m_g \left(\frac{\partial W}{\partial z_A} + \sum_i \hat{z}_i \frac{\partial^2 W}{\partial z_A \partial \hat{z}_i} + (A-3) \frac{\partial W}{\partial z_A} \right) + m_g^2 \left(z_A^* + \frac{\partial f}{\partial z_A} \right) = 0 \tag{3.16}$$

which may be written as,

$$\sum_{B,C} \left(\frac{\partial^2 \tilde{W}}{\partial z_A \partial z_C} + m_g \frac{\partial g_C}{\partial z_A} \right) (I+h)^{-1}_{BC} \left(\frac{\partial \tilde{W}}{\partial z_B} + m_g g_B \right)^*$$

$$= -\sum_{\alpha, B} (I+h)^{-1}_{B\alpha} \left(\frac{\partial^2 \tilde{W}}{\partial z_A \partial z_\alpha} + m_g \frac{\partial g_\alpha}{\partial z_A} \right) \left(\frac{\partial \tilde{W}}{\partial z_B} + m_g g_B \right)^*$$

$$- \sum_{B,\alpha} (I+h)^{-1}_{\alpha B} \left(\frac{\partial^2 \tilde{W}}{\partial z_A \partial z_B} + m_g \frac{\partial g_B}{\partial z_A} \right) \left(\frac{\partial \tilde{W}}{\partial z_\alpha} + m_g g_\alpha \right)^*$$

$$- \sum_{\alpha, \beta} (I+h)^{-1}_{\alpha\beta} \left(\frac{\partial^2 \tilde{W}}{\partial z_A \partial z_\beta} + m_g \frac{\partial g_\beta}{\partial z_A} \right) \left(\frac{\partial \tilde{W}}{\partial z_\alpha} + m_g g_\alpha \right)^*$$

$$- m_g \sum_{i,j} \frac{\partial g_i^*}{\partial z_A} (I+h)^{-1}_{ij} \left(\frac{\partial \tilde{W}}{\partial z_j} + m_g g_j \right)$$

$$- \sum_{i,j} \frac{\partial}{\partial z_A} \left\{ (I+h)^{-1}_{ij} \right\} \left(\frac{\partial \tilde{W}}{\partial z_i} + m_g g_i \right)^* \left(\frac{\partial \tilde{W}}{\partial z_j} + m_g g_j \right)$$

$$- m_g \left(\frac{\partial W}{\partial \hat{z}_A} + \sum_i \hat{z}_i \frac{\partial^2 W}{\partial z_A \partial \hat{z}_i} + (A-3) \frac{\partial W}{\partial z_A} \right)$$

$$- m_g^2 \left(z_A^* + \frac{\partial f}{\partial z_A} \right) \tag{3.17}$$

We may solve these equations to find z_A as a function of the fields z_α and substitute these values in (3.14) in order to find the effective potential as a function of z_α.

We solve these equations by iteration as in Sec.II. We start from the ansatz,

$$\frac{\partial \widetilde{W}}{\partial z_A} + m_g g_A \sim m_g^2 \qquad \forall A \tag{3.18}$$

Using Eqs.(2.3), (3.7), (3.15) and (3.18) we may show that the right hand side of (3.17) is of order m_g^2M, the terms of order m_g^2M being given by,

$$-\sum_{B,\alpha} (I+h)^{-1}_{\alpha B} \; M_{AB} \; \left(\frac{\partial \widetilde{W}}{\partial z_\alpha} + m_g g_\alpha \right)^* + m_g^2 \, (z_A^{(0)*} + g_A)(A-2)$$

$$- m_g \sum_B \hat{z}_B \, M_{AB} \; - m_g^2 \left(z_A^{(0)*} + \frac{\partial f}{\partial z_A} \right) \equiv m_g^2 \, y_A$$

$$\tag{3.19}$$

In deriving the above equation we have used the relation,

$$\frac{\partial W}{\partial z_A} = \frac{\partial \widetilde{W}}{\partial z_A} - m_g z_A^{(0)*} = -m_g \, (z_A^{(0)*} + g_A) + O(m_g^2)$$

$$\tag{3.20}$$

Eq.(3.17) now gives us,

$$\sum_{B,C} M_{AC} \; (I+h)^{-1}_{BC} \; \left(\frac{\partial \widetilde{W}}{\partial z_B} + m_g g_B \right)^*$$

$$= m_g^2 \, y_A + O(m_g^2) \tag{3.21}$$

which may be solved as,

$$\left(\frac{\partial \tilde{W}}{\partial z_B} + m_g\, g_B \right)^* = m_g^2 \left[(I+h)^{-1} M \right]^{-1}_{AB} \; y_A \tag{3.22}$$

$[(I+h)^{-1}M]_{AB}$ is a non-singular matrix[13] with eigenvalues of order M. Hence the right hand side of the equation is of order m_g^2, which shows the validity of the ansatz (3.18). We may now try to solve Eq.(3.22) by expanding z_A as in Eq.(2.8). Eq.(3.22) gives,

$$\begin{aligned}
&\left(\frac{\partial^2 W}{\partial z_B \partial z_A}\right)_0 \left(z_A^{(1)} + z_A^{(2)} \right) + \frac{1}{2}\left(\frac{\partial^3 W}{\partial z_B\, \partial z_A\, \partial z_C}\right) z_A^{(1)} z_C^{(1)} \\
&+ \left(\frac{\partial^3 W}{\partial z_B\, \partial z_A\, \partial z_\alpha}\right) z_A^{(1)} z_\alpha + \frac{1}{2}\left(\frac{\partial^3 W}{\partial z_B\, \partial z_\alpha\, \partial z_\beta}\right) z_\alpha z_\beta \\
&+ m_g \left(z_B^{(0)*} + z_B^{(c)*} - z_B^{(0)*} \right) + m_g \left(g_B(z_i^{(0)}) + g_B(z_A^{(0)} + z_A^{(1)}, z_\alpha) - g_B(z_i^{(0)}) \right) \\
&= m_g^2 \left[(I+h)^{-1} M \right]^{-1}_{AB} \; y_A + O(m_g^3/M)
\end{aligned} \tag{3.23}$$

which gives,

$$z_A^{(1)} = -m_g (M^{-1})_{AB} \left(z_B^{(0)*} + g_B(z^{(0)}) \right) \tag{3.24}$$

$$z_A^{(2)} = (M^{-1})_{AB} \; [\; m_g{}^2\{(I+h)^{-1} M\}^{-1}_{CB} \; y_C - \frac{1}{2}\left(\frac{\partial^3 W}{\partial z_B \, \partial z_C \partial z_D}\right) z_C^{(1)} z_D^{(1)}$$

$$-\left(\frac{\partial^3 W}{\partial z_B \, \partial z_C \, \partial z_\alpha}\right) \; z_C^{(1)} \, z_\alpha - \frac{1}{2}\left(\frac{\partial^3 W}{\partial z_B \; \partial z_\alpha \, \partial z_\beta}\right) \; z_\alpha \, z_\beta \; - m_g (z_B^{(c)*} - z_B^{(0)*})$$

$$- m_g \; (g_B \; (z_A^{(0)} + z_A^{(1)} \, , \; z_\alpha) \; - g_B \; (z_i^{(0)}))] \qquad (3.25)$$

Eq.(3.24) shows that $z_A^{(1)}$ is independent of the light fields z_α. Thus $\hat{z}_B^{(1)} = z_B^{(1)} - z_B^{(c)}$ must also be independent of z_α. Thus the only z_α dependent term in y_A to order M comes from the first term on the left hand side of Eq.(3.19). As in Sec.II, only the z_α dependent part of $z_A^{(2)}$ is relevant in calculating the effective potential to order $m_g{}^4$. Hence we may now substitute the values of $z_A^{(1)}$ and the z_α dependent part of $z_A^{(2)}$ in (3.14) and calculate the effective potential as in Sec.II. Here we shall show that this effective potential is at most of order $m_g{}^4$ for $z_\alpha \sim m_g$, so that the tree level hierarchy is not destroyed by radiative corrections. As can be seen from (3.22), $(\partial\tilde{W}/\partial z_B + m_g g_B)$ is of order $m_g{}^2$, hence the first term on the right hand side of (3.14) is of order $m_g{}^4$. Contribution from other terms to order $m_g{}^3 M$ may be obtained by expanding various terms around the point $z_i = z_i^{(0)}$. This contribution may be written as,

$$m_g \; (A-3) \;\; \{ \; \frac{1}{2}\left(\frac{\partial^2 W}{\partial z_A \, \partial z_B}\right)_0 \; z_A^{(1)} \;\; z_B^{(1)} \;\; + \; h.c. \; \} - \{ m_g{}^2 \, \hat{z}_A^{(1)} \; (z_A^{(0)*} + g_A (z^{(0)}) + h.c.$$

$$+ \; m_g{}^2 \;\; [\, | z_A^{(0)} |^2 \; + \; f \, (z^{(0)}) \; + \; (z_A^{(0)*} \, z_A^{(1)} \; + h.c.) \; + (\left(\frac{\partial f}{\partial z_A}\right)_0 z_A^{(1)} + h.c.)]$$

$$+ m_g (A-3) \{ W (z^{(0)}) + h.c. \} + \;\; O \, (m_g{}^4) \qquad (3.26)$$

However, since $z_A^{(1)}$ is independent of the light fields z_α, each of the terms in (3.26) is independent of z_α. Hence the effective potential involving the field z_α is at most of order m_g^4, showing that the radiative corrections do not destroy the tree level mass hierarchy.

Note that the absence of light singlets is crucial in our analysis. If some of the fields z_α are singlets of the group H, the corresponding g_α and $\partial f/\partial z_\alpha$ may be of order M. This will give contribution of order $m_g^3 M$ or more to the effective potential from the $|\partial\tilde{W}/\partial z_\alpha + m_g g_\alpha|^2$ term, as well as the $(\partial f/\partial z_\alpha) z_\alpha$ term in the Taylor series expansion of f.

Finally we shall comment on the terms of order $\tilde{F}^3$ in (3.13). We may include the effect of these terms by solving for $\tilde{F}$ iteratively, the $O(\tilde{F}^3)$ terms being ignored at the first stage of iteration. This gives a solution $\tilde{F}_i \sim m_g^2$, as is seen from Eq.(3.18) and a corresponding expression for $\partial\tilde{W}/\partial z_\alpha + m_g g_\alpha$. If we substitute this value of $\tilde{F}$ in the next stage of iteration, the order $\tilde{F}^3$ terms contribute at most to $O(m_g^2)$ in the new value of $\tilde{F}$ and $O(m_g^4)$ to the potential. Also, since $\partial\tilde{F}_i/\partial z_j \lesssim M$, the derivative of this new term with respect to z_A contribute at most a term of order $m_g^2 M$ on the right hand side of Eq.(3.17). Hence it does not upset any of the results obtained at the first stage of iteration, and the mass hierarchy is maintained.

IV. CONCLUSION

In this paper we have studied radiative corrections in a supersymmetric theory with soft breaking terms induced by N=1 supergravity. We consider models where the low energy theory is invariant under some group H and none of the light fields transform as a singlet of this group. Also in this paper we consider theories without any gauge fields. For this class of models, any hierarchy of mass scales, present in the limit of unbroken supersymmetry, is shown to be preserved after the inclusion of the supersymmetry breaking terms at the tree level, and also after inclusion of radiative corrections to all orders in the perturbation theory.

Although the absence of gauge fields and light singlet fields is a sufficient condition for the stability of mass hierarchy, it is, by no means, a necessary condition. However, the analysis becomes considerably more complicated in these cases. In particular, when gauge fields are present, the theorem of Ref.9, as expressed in Eq.(3.11), is no longer sufficient to prove the stability of the mass hierarchy. We must also use the supersymmetric Slavnov-Taylor identities[14] of gauge invariance in order to restrict the possible form of the effective action. Work towards this direction is in progress.

The effect of light singlets in grand unified theories based on N=1 supergravity has been discussed by Nilles, Srednicki and Wyler and by Lahanas[7]. They showed that the presence of trilinear coupling between one light singlet of mass of order m_g and two heavy fields of mass of order M in the superpotential destroys the mass hierarchy. This rules out the use of sliding singlet mechanism[15] to keep the weak doublet higgs light compared to its color triplet partner in SU(5) grand unified theories. The problem may be avoided in a class of SU(6) grand unified theories in which the sliding singlet acquires a mass of order $\sqrt{m_g M}$[16]. In these models, the problems indicated in Ref.7 are absent, although an all order proof of the stability of mass hierarchy is still lacking.

Finally we should mention that in the analysis given in this paper, we have ignored the terms in the tree level Lagrangian which have explicit powers of M_p in the denominator (except those which come in the combination $\mu^3/M_p^2 \sim m_g$). If we want to include these terms in our analysis, we must also include the effect of loop corrections involving the gravitons and the gravitinos for consistency. At present there is no known way to take these corrections into account, since the N=1 supergravity theories are neither renormalizable nor finite. Hence for the time being we have to be satisfied with the fact that in the zeroth order term in the expansion in powers of M_p^{-1}, the radiative corrections do not destroy the mass hierarchy to all orders in the loop expansion.

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[6]The origin of this coincidence may be traced to the following reason. It was shown in Ref.5 that the value of z_h at the minimum of the full potential for arbitrary values of order m_g of the light fields z_α is equal to $z_h^{(0)}$ up to correction terms that are irrelevant for evaluating the effective potential to order m_g^4. Hence it is not surprising that the answer that we get by setting $z_h=z_h^{(0)}$ from the very beginning, coincides with the more detailed analysis of Ref.5.

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[11] We can remove extra powers of θ and/or $\bar{\theta}$ coming from extra powers of $\tilde{F}$, $\tilde{F}^{\dagger}$, η or $\eta^{\dagger}$ by using the covariant derivative operators D and $\bar{D}$ acting on these fields, so as to get an integral of the form $\int d^4\theta\ \theta^2\bar{\theta}^2$ from (3.11).

[12] The absence of such power law infrared divergences may be seen in the following way. The singularity of a diagram in the $m_g \to 0$ limit comes from the singularities of the internal propagarors. Only those propagators which have masses of order m_g and which carry small internal momenta of order m_g contribute to the singular part. Assuming that the scales of two and three point couplings between the fields of mass of order m_g are set by the mass m_g, and not the large mass M, we may show by simple power counting that any 1PI graph,

the total mass dimension of whose external lines is less than five, is at most logarithmically divergent in the $m_g \to 0$ limit. (We must count η as well as each insertion of the $\partial/\partial z_i$ operator as an external line of mass dimension unity in this analysis). Thus, if in the n loop order, the effective action involving the light fields is independent of the heavy scale M, then in the n+1 loop order the functions f, g_i, h_{ij}, $\partial f/\partial z_i$ and $\partial g_i/\partial z_j$ satisfy the constraints mentioned above. This result may then be used to show that in the n+1 loop order the effective action involving the light fields is independent of the heavy scale M.

[13]At this point we should remind the reader that in calculating $(I+h)^{-1}$ from $(I+h)$ we consider $(I+h)$ as a matrix in the space labelled by the index i, which includes the heavy, as well as the light fields. On the other hand, in calculating $[(I+h)^{-1}M]^{-1}$ in Eq.(3.23), we consider $(I+h)^{-1}$ and M as matrices in the space spanned by the index A, which includes only the heavy fields. Hence this expression is not identical to $M^{-1}(I+h)$.

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